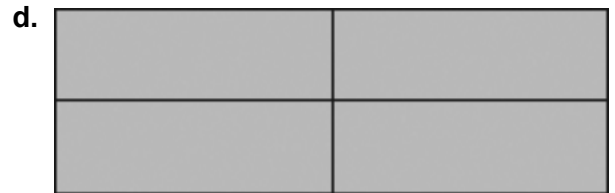
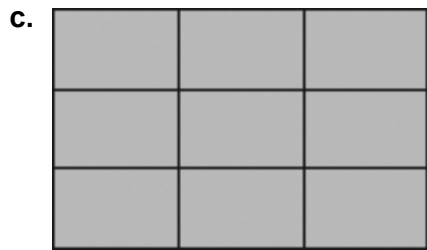
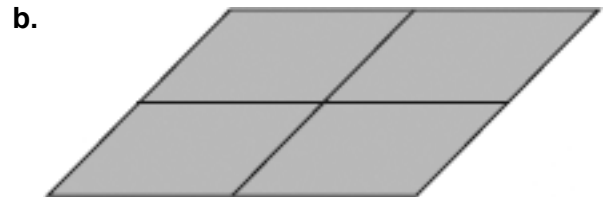
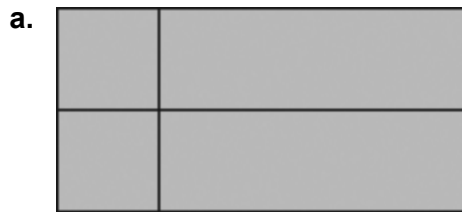


Applications

1. Look for rep-tile patterns in the designs below. For each design,
 - Decide whether the small quadrilaterals are similar to the large quadrilateral. Explain.
 - If the quadrilaterals are similar, give the scale factor from each small quadrilateral to the large quadrilateral.



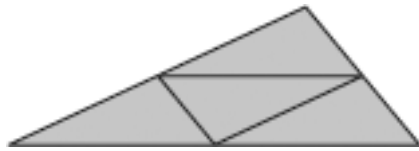
2. Suppose you put together nine copies of a rectangle to make a larger, similar rectangle.
 - a. How is the area of the larger rectangle related to the area of the smaller rectangle?
 - b. What is the scale factor from the smaller rectangle to the larger rectangle?

3. Suppose you divide a rectangle into 25 smaller rectangles such that each rectangle is similar to the original rectangle.
 - a. How is the area of each of the smaller rectangles related to the area of the original rectangle?
 - b. What is the scale factor from the original rectangle to each of the smaller rectangles?

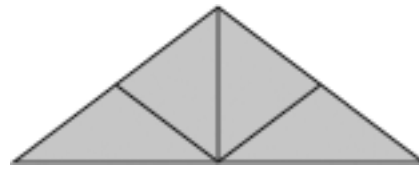
4. Look for rep-tile patterns in the figures below.

- Tell whether the small triangles are similar to the large triangle. Explain.
- If the triangles are similar, give the scale factor from each small triangle to the large triangle.

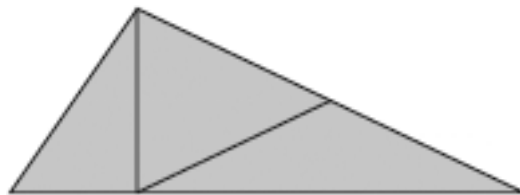
a.



b.



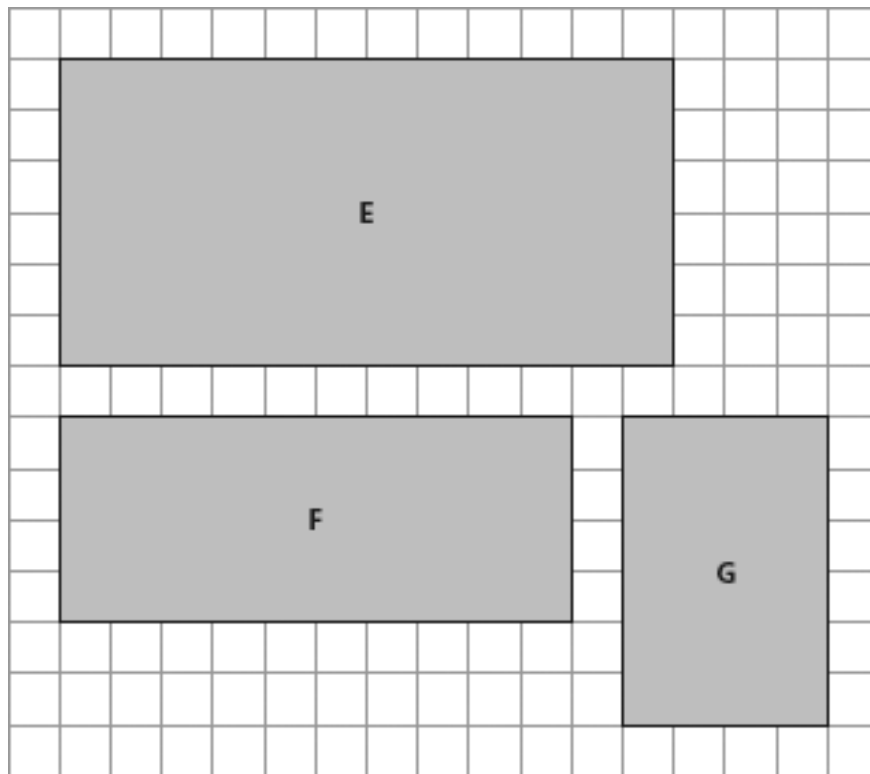
c.



d.

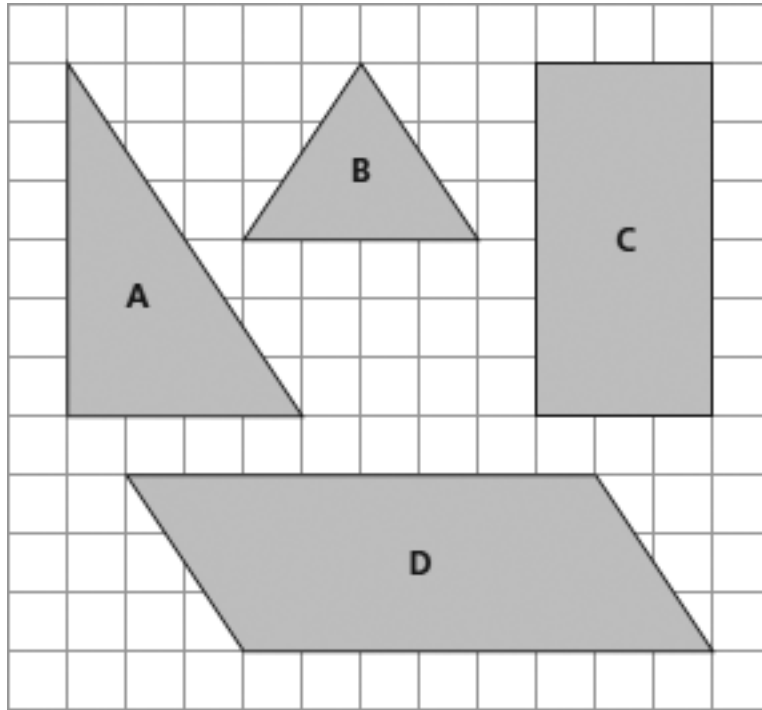


5. a. For rectangles E–G, give the length and width of a different, similar rectangle. Explain how you know the new rectangles are similar.

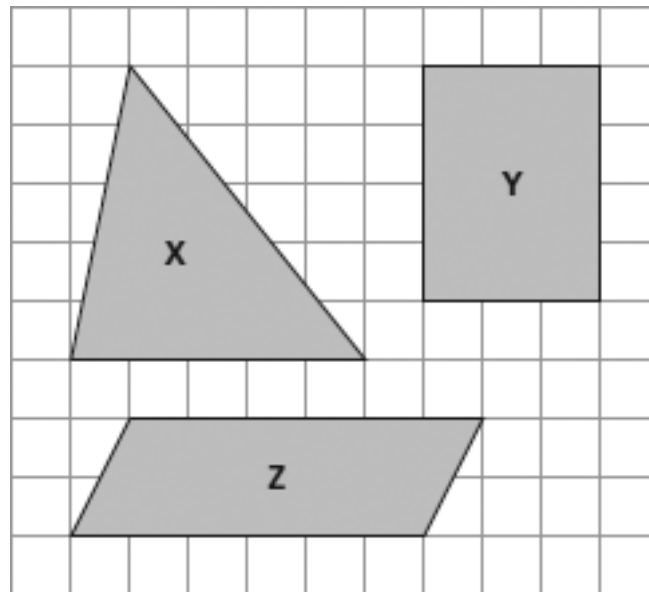


b. Give the scale factor from each original rectangle in part (a) to the similar rectangles you described. Explain what the scale factor tells you about the corresponding lengths, perimeters, and areas.

6. Copy polygons A–D onto grid paper. Draw line segments that divide each of the polygons into four congruent polygons that are similar to the original polygon.

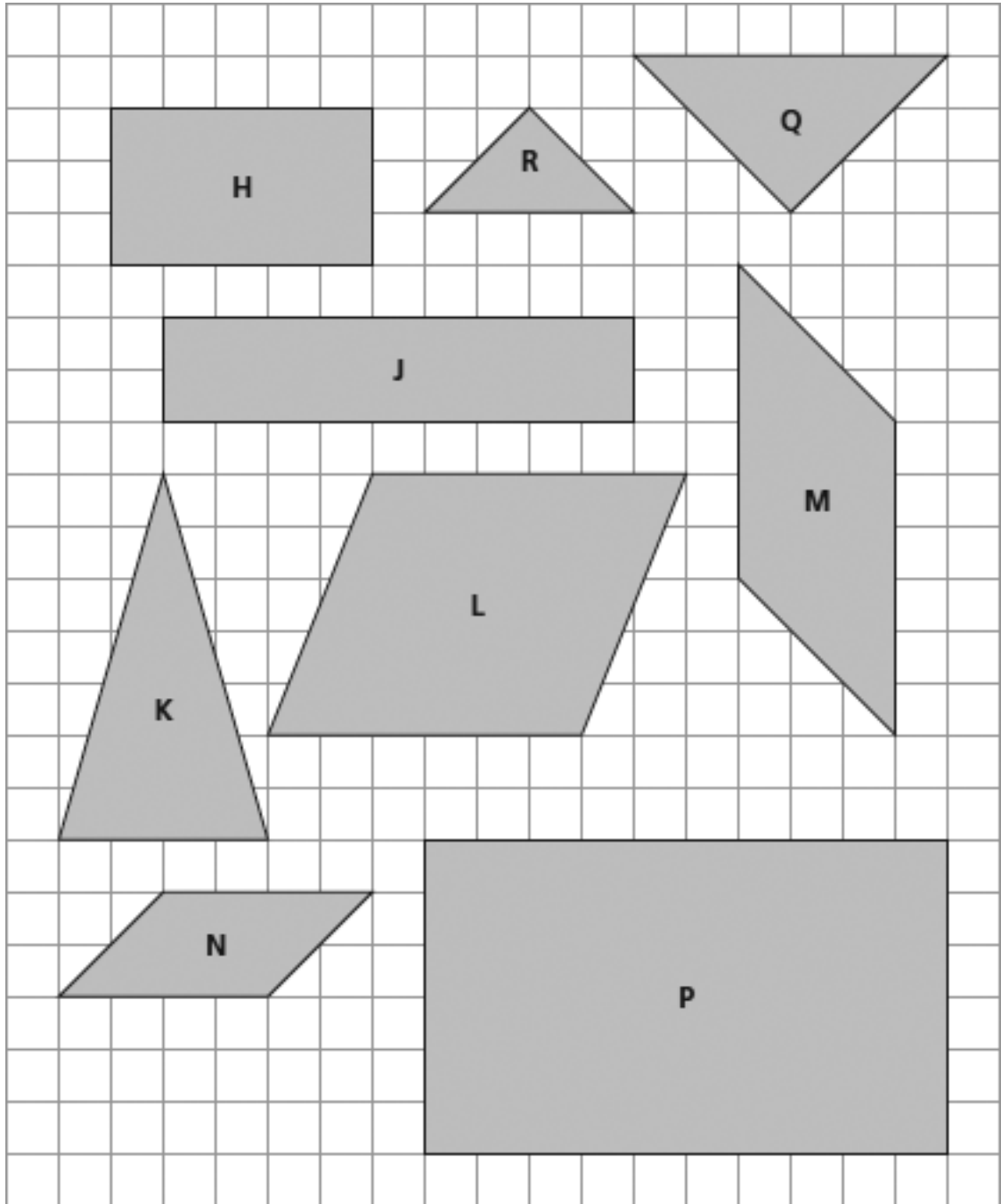


7. For parts (a)–(c), use grid paper.



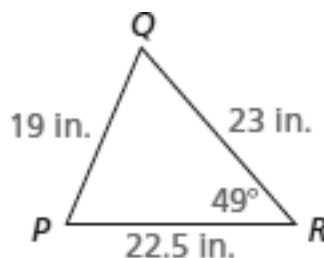
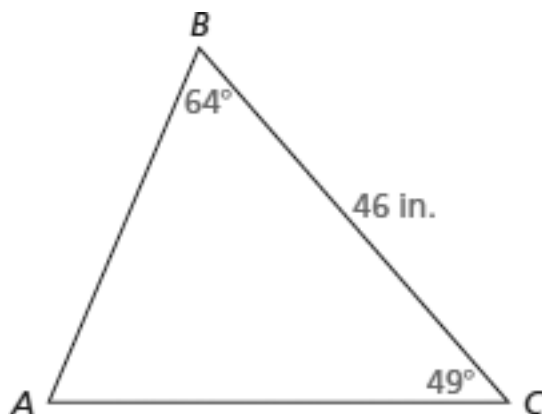
- Sketch a triangle similar to Triangle X with an area that is $\frac{1}{4}$ the area of Triangle X.
- Sketch a rectangle similar to Rectangle Y with a perimeter that is 0.5 times the perimeter of Rectangle Y.
- Sketch a parallelogram similar to Parallelogram Z with side lengths that are 1.5 times the side lengths of Parallelogram Z.

8. Use the polygons below.



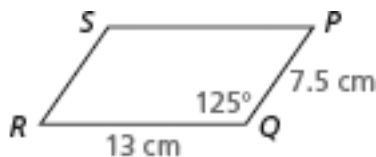
- a. List pairs of similar shapes.
- b. For each pair of similar shapes, find the scale factor from the smaller shape to the larger shape.

Triangle ABC is similar to triangle PQR . For Exercises 9–14, find the indicated angle measure or side length.



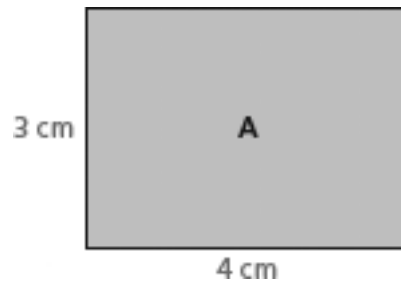
- | | |
|-------------------------|---------------------------------|
| 9. angle A | 10. angle Q |
| 11. angle P | 12. length of side AB |
| 13. length of side AC | 14. perimeter of triangle ABC |

Multiple Choice For Exercises 15–18, use the similar parallelograms below.

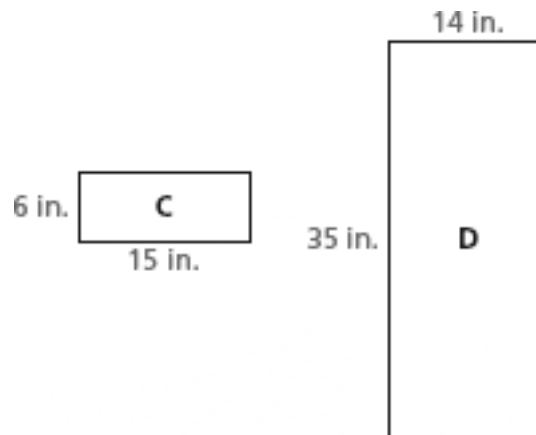


- | | | | | |
|--|----------------------|--------------------|--------------------|--------------------|
| 15. What is the measure of angle D ? | A. 55° | B. 97.5° | C. 125° | D. 135° |
| 16. What is the measure of angle R ? | F. 55° | G. 97.5° | H. 125° | J. 135° |
| 17. What is the measure of angle S ? | A. 55° | B. 97.5° | C. 125° | D. 135° |
| 18. What is length of side AB ? | F. 3.75 cm | G. 13 cm | H. 15 cm | J. 26 cm |

19. Suppose Rectangle B is similar to Rectangle A below. The scale factor from Rectangle A to Rectangle B is 4. What is the area of Rectangle B?



20. Suppose Rectangle E has an area of 9 square centimeters and Rectangle F has an area of 900 square centimeters. The two rectangles are similar. What is the scale factor from Rectangle E to Rectangle F?
21. Suppose Rectangles X and Y are similar. Rectangle X is 5 centimeters by 7 centimeters. The area of Rectangle Y is 140 square centimeters. What are the dimensions of Rectangle Y?
22. Anya and Jalen disagree about whether the two figures below are similar. Do you agree with Anya or with Jalen? Explain.



Anya's Reasoning

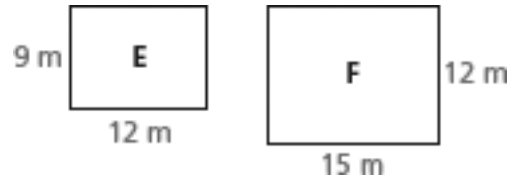
The two rectangles are not similar. The height of Rectangle D is almost 6 times the height of Rectangle C, but the widths are almost the same. Similar rectangles must have the same scale factor for the base and the height.



Jalen's Reasoning

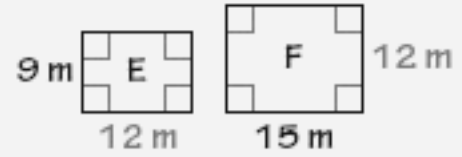
The two rectangles are similar. The scale factor from C to D is $\frac{7}{3}$. You can multiply the short side of C (the height) by to get 14 inches, which is the short side of D (the base). This scale factor also works for the long sides of the rectangles since $15 \times \frac{7}{3} = 35$.

23. Evan, Melanie, and Wyatt discuss whether the two figures at the right are similar. Do you agree with Evan, Melanie, or Wyatt? Explain.



Evan's Reasoning

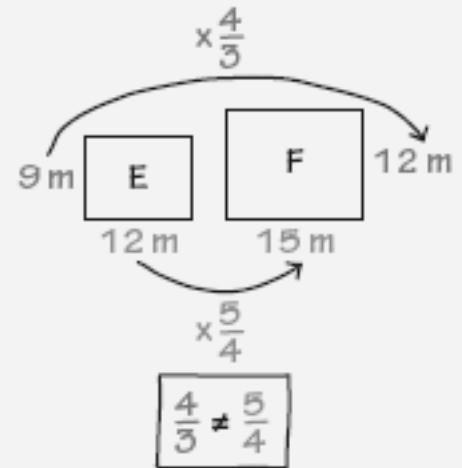
Rectangles E and F are similar because each shape has four right angles. Also, each rectangle has at least one side that is 12 meters long.



Melanie's Reasoning

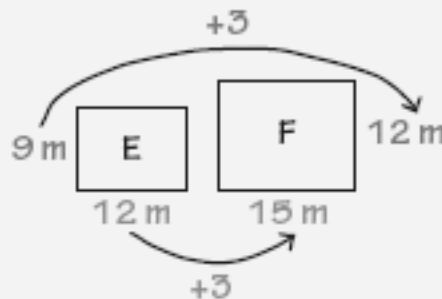
The scale factor for the height from rectangle E to rectangle F is $\frac{12}{9}$, or $\frac{4}{3}$.

The scale factor for the base is $\frac{15}{12}$, or $\frac{5}{4}$. $\frac{4}{3} \neq \frac{5}{4}$, so the rectangles are not similar.

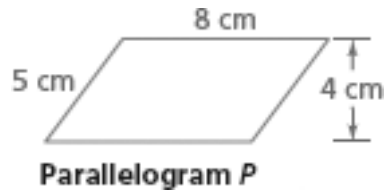


Wyatt's Reasoning

Rectangles E and F are similar. Rectangle F is 3 meters taller than Rectangle E since $9 \text{ meters} + 3 \text{ meters} = 12 \text{ meters}$. Rectangle F is also 3 meters wider than Rectangle E since $12 \text{ meters} + 3 \text{ meters} = 15 \text{ meters}$. Each dimension of Rectangle F is 3 meters greater than the corresponding dimension of Rectangle E, so the rectangles are similar.



24. Janine, Trisha, and Jeff drew parallelograms that are similar to Parallelogram P below.



Each student claims that the scale factor from P to the sketched parallelogram is 4. Are any of the students correct in their reasoning? Explain.

Janine's Method

I divided the original parallelogram into four similar parallelograms. Parallelogram P is four times as large as each of the new parallelograms.



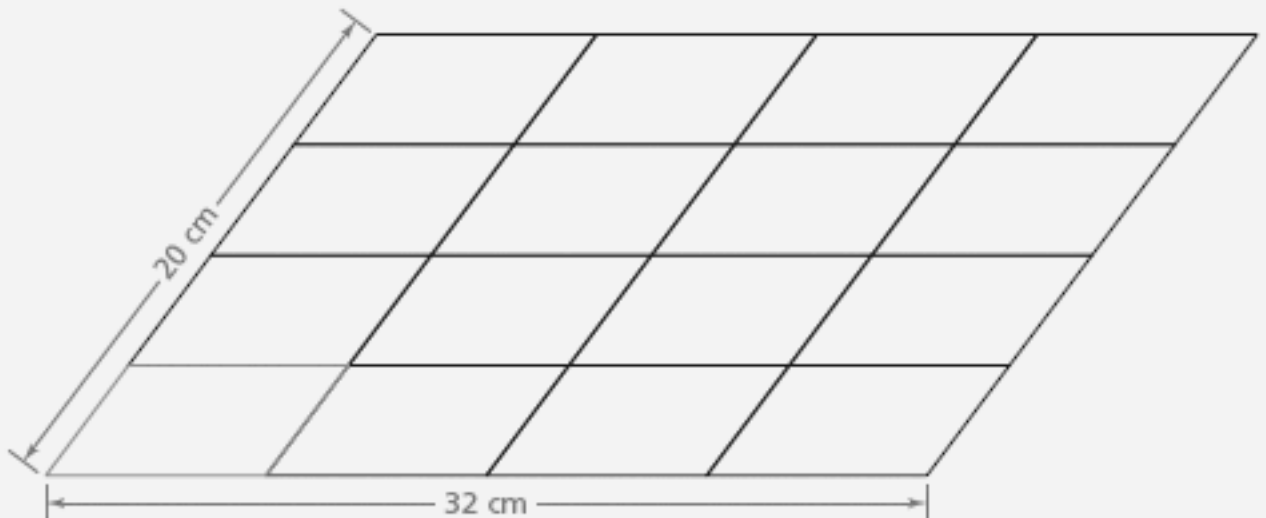
Trisha's Method

I sketched four copies of parallelogram P . The shape has four times the area of parallelogram P .

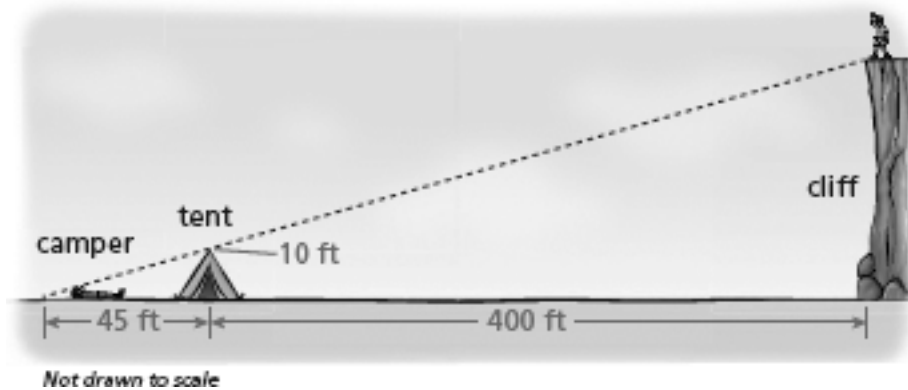


Jeff's Method

I wanted a scale factor of 4. The perimeter of the original shape is 26 centimeters. I drew a parallelogram with a perimeter of 4×26 centimeters = 104 centimeters.

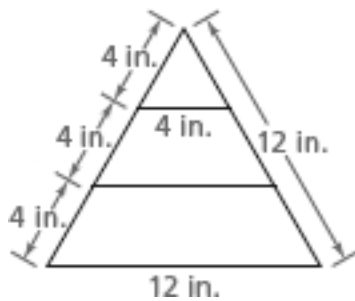


25. Judy lies on the ground 45 feet from her tent. Both the top of the tent and the top of a tall cliff are in her line of sight. Her tent is 10 feet tall. About how high is the cliff? Assume the two triangles are similar.

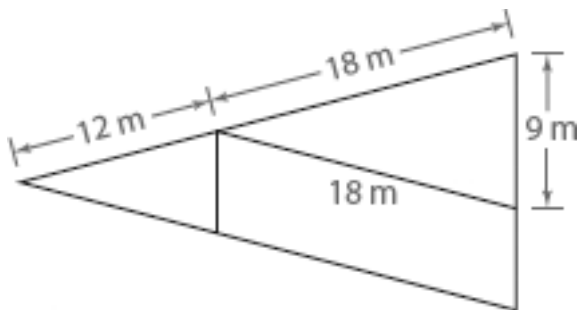


For Exercises 26–28, each triangle has been subdivided into triangles that are similar to the original triangle. Copy each triangle and label as many side lengths as you can.

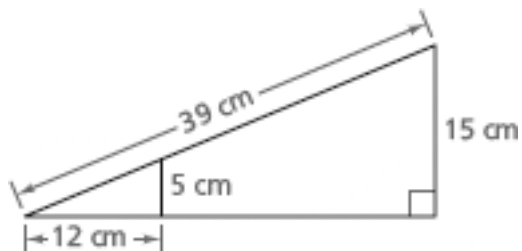
26.



27.

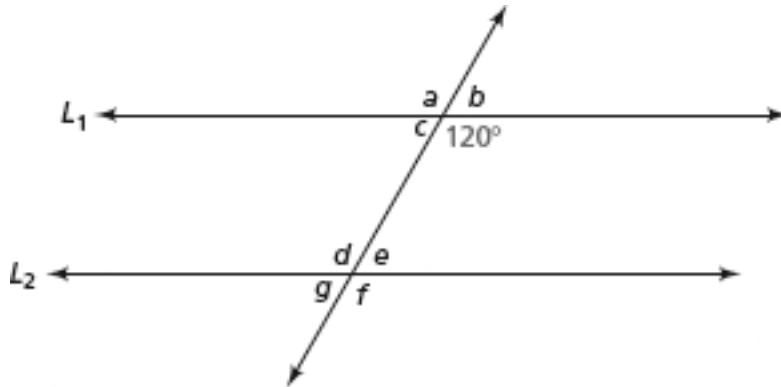


28.

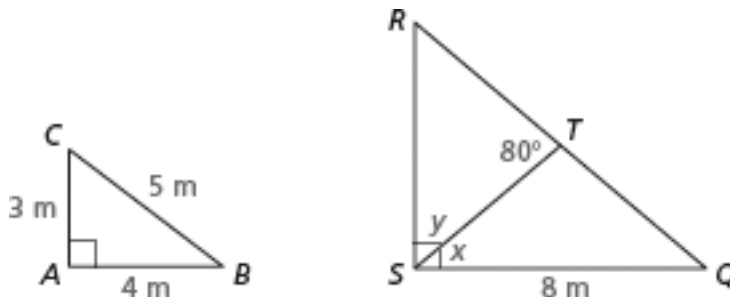


Connections

29. In the figure below, lines L_1 and L_2 are parallel.
- a. Use what you know about parallel lines to find the measures of angles a through g .



- b. List all pairs of *supplementary* angles in the diagram.
30. For each of the following angle measures, find the measure of its supplementary angle.
- a. 160° b. 90° c. x°
31. The right triangles below are similar.



- a. Find the length of side RS .
- b. Find the length of side RQ .
- c. The measure of angle x is about 40° . If the measure of angle x were exactly 40° , what would be the measure of angle y ?
- d. Use your answer from part (c) to find the measure of angle R . Explain how you can find the measure of angle C .
- e. Angle x and angle y are *complementary angles*. Find two additional pairs of complementary angles in Triangles ABC and QRS .

32. For parts (a)–(f), find the number that makes the fractions equivalent.

a. $\frac{1}{2} = \frac{3}{\square}$

b. $\frac{5}{6} = \frac{\square}{24}$

c. $\frac{3}{4} = \frac{6}{\square}$

d. $\frac{8}{12} = \frac{2}{\square}$

e. $\frac{3}{5} = \frac{\square}{100}$

f. $\frac{6}{4} = \frac{\square}{10}$

33. For parts (a)–(f), suppose you copy a figure on a copier using the given scale factor. Find the scale factor from the original figure to the copy in decimal form.

a. 200%

b. 50%

c. 150%

d. 125%

e. 75%

f. 25%

34. Write each fraction as a decimal and as a percent.

a. $\frac{2}{5}$

b. $\frac{3}{4}$

c. $\frac{3}{10}$

d. $\frac{1}{4}$

e. $\frac{7}{10}$

f. $\frac{7}{20}$

g. $\frac{4}{5}$

h. $\frac{7}{8}$

i. $\frac{15}{20}$

j. $\frac{3}{5}$

35. For parts (a)–(d), tell whether the figures are mathematically similar. Explain your reasoning. If the figures are similar, give the scale factor from the left figure to the right figure.

a.



b.



c.

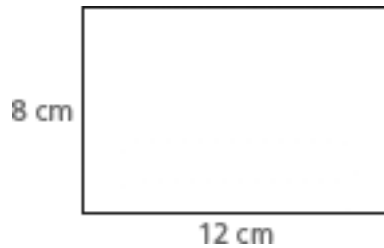


d.

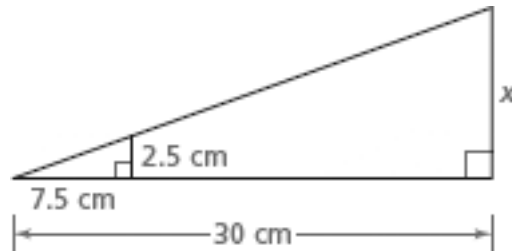


For Exercises 36–38, decide whether the statement is true or false. Explain your reasoning.

36. All squares are similar.
37. All rectangles are similar.
38. If the scale factor between two similar shapes is 1, then the two shapes are the same size.
39. a. Suppose the following rectangle is reduced by a scale factor of 50%. What are the dimensions of the reduced rectangle?



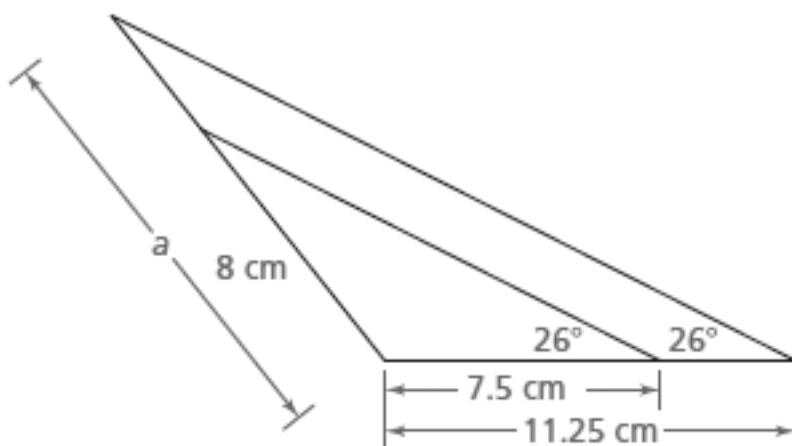
- b. Suppose the reduced rectangle from part (a) is reduced again by a scale factor of 50%. What are the dimensions of the new rectangle? Explain your reasoning.
- c. How does the reduced rectangle from part (b) compare to the original rectangle from part (a)?
40. **Multiple Choice** What is the value of x ? The diagram is not to scale.



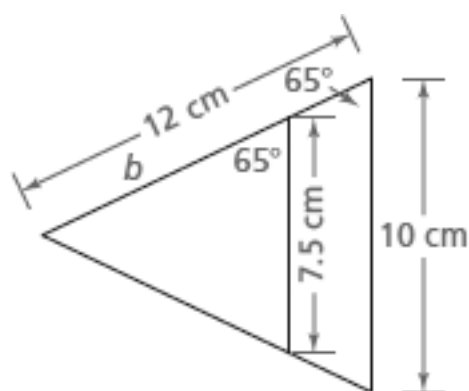
- A. 3 cm B. 10 cm
- C. 12 cm D. 90 cm

For Exercises 41 and 42, find the missing side length. The diagrams are not to scale.

41.

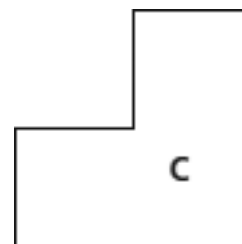
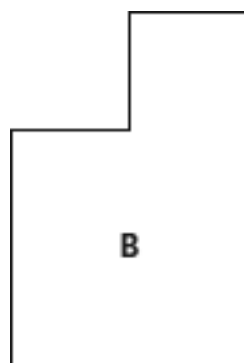
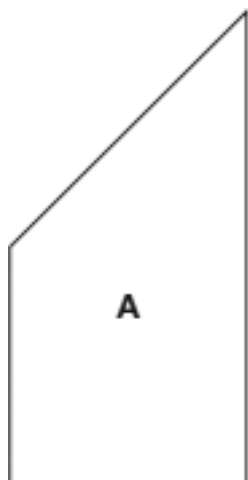


42.



Extensions

43. Trace each shape. Divide each shape into four smaller, identical pieces that are similar to the original shape.



- 44.** The **midpoint** of a line segment is a point that divides the segment into two segments of equal length. Draw a figure on grid paper by following these steps:
- Step 1:** Draw a large square.
 - Step 2:** Mark the midpoint of each side.
 - Step 3:** Connect the midpoints, in order, with four line segments to form a new figure. (The line segments should not intersect inside the square.)
 - Step 4:** Repeat Steps 2 and 3 three more times. Work with the newest figure each time.
- a. What kind of figure is formed when the midpoints of the sides of a square are connected?
 - b. Find the area of the original square you drew in Step 1.
 - c. Find the area of each of the new figures that was formed.
 - d. How do the areas change between successive figures?
 - e. Are there any similar figures in your final drawing? Explain.
- 45.** Repeat Exercise 44 starting with an equilateral triangle, connecting three line segments to form a new triangle each time.
- 46.** Suppose Rectangle A is similar to Rectangle B and to Rectangle C. Can you conclude that Rectangle B is similar to Rectangle C? Explain. Use drawings and examples to illustrate your answer.

47. You can subdivide figures to get smaller figures that are mathematically similar to the original. The mathematician Benoit Mandelbrot called these figures *fractals*. A famous example is the Sierpinski triangle.

Sierpinski Triangle

You can follow these steps to make the Sierpinski triangle.



Step 1: Draw a triangle.
(It does not have to be an equilateral triangle.)



Step 2: Mark the midpoint of each side. Connect the midpoints to form four identical triangles that are similar to the original. Shade the center triangle.



Step 3: For each unshaded triangle, mark the midpoints. Connect them in order to form four identical triangles. Shade the center triangle in each case.



Step 4: Repeat Steps 2 and 3 over and over. To make a real Sierpinski triangle, you need to repeat the process an infinite number of times! This triangle shows five subdivisions.

- Follow the steps for making the Sierpinski triangle until you subdivide the original triangle three times.
- Describe any patterns you observe in your figure.
- Mandelbrot used the term *self-similar* to describe fractals like the Sierpinski triangle. What do you think this term means?

Use the paragraph below for Exercises 48–52.

When you find the area of a square, you multiply the length of one side by itself. For a square with a side length of 3 units, you multiply 3×3 to get 9 square units. For this reason, mathematicians call 9 the *square* of 3.

The *square root* of 9 is 3. The symbol $\sqrt{\quad}$ is used for the square root. This gives the fact family below.

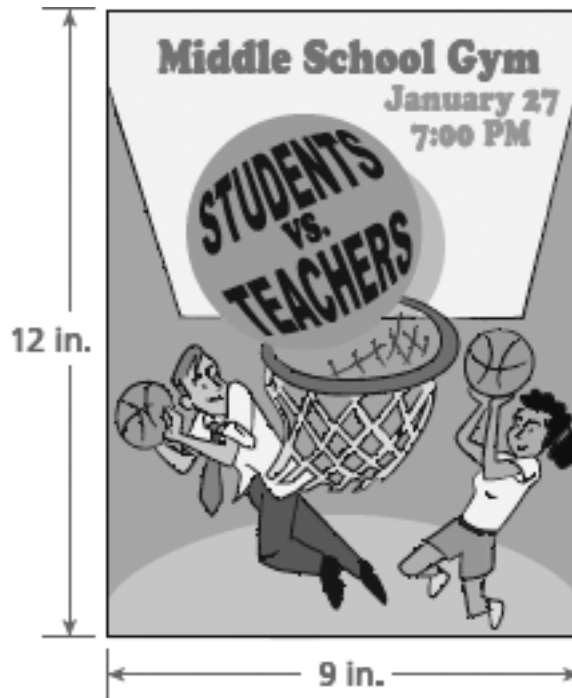
$$3^2 = 9$$
$$\sqrt{9} = 3$$

- 48.** The square below has an area of 10 square units. Write the side length of this square using a square root symbol.



- 49. Multiple Choice** What is the square root of 144?
F. 7 **G.** 12 **H.** 72 **J.** 20,736
- 50.** What is the side length of a square with an area of 144 square units?
- 51.** You have learned that if a figure grows by a scale factor of s , the area of the figure grows by a factor of s^2 . If the area of a figure grows by a factor of f , what is the scale factor?
- 52.** Find three examples of squares and square roots in the work you have done so far in this Unit.

53. Song makes a copy of the poster below.



- She presses the 50% button on the copy machine. Now the length and width of the poster are each half of their original sizes. Song thinks that if she enlarges the copy by 150%, the new copy will be the same size as the original. Is she correct?
- Suppose Song had done the opposite in part (a), first enlarging the poster by 150%, and then reducing the copy by 50%. Will the final copy be the same size as the original? Will it be the same size as the copy made in part (a)?
- Song uses the same process from parts (a) and (b) with a different-sized poster. Does she get similar results?
- Song applied a scale factor of 25% to shrink the original poster. Now she wants to get the poster back to the original size. What scale factor should she use? Explain your reasoning.
- Suppose Song had used 75% and 125% in parts (a) and (b) instead of 50% and 150%. What would have happened?
- What general statements can you make about applying any pair of two scale factors one after the other? Consider a pair of two enlargements, a pair of two reductions, and a pair consisting of one enlargement and one reduction.